AN IMPROVED ALGORITHM FOR THE MACHINE SCHEDULING PROBLEM WITH JOB DELIVERY COORDINATION

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Abstract

A two-stage supply chain scheduling problem is considered, where the first stage is job production and the second stage is job delivery. The focus is on the study of the integration of production scheduling with delivery of finished products to customers. In our considered model each job can be processed on either of two identical machines, and then delivered by a vehicle to a customer location. We present an improved algorithm with the worst-case performance ratio $\frac{14}{9} + \varepsilon$, which improves the known upper bounds of 2 and $\frac{5}{3}$ in [Chang, Y.C. and Lee, C.Y., E. J. O. R., 158 (2004), pp. 470-487; Zhong, W., Dosa, G. and Tan, Z.Y., E. J. O. R., 182(2007), pp. 1057-1072].

1 Introduction

The problem we considered was firstly proposed by Chang and Lee [1], and can be described as follows: there are n job $N = \{J_1, J_2, \cdots, J_n\}$ which must be processed on two identical parallel machines and then are delivered to a customer. Job J_j has a processing time p_j and a size s_j. There is only one vehicle and its capacity is z. All jobs delivered together in one vehicle are defined as a delivery batch. We define the value of a schedule σ, denoted by C_{max}, as the time when the vehicle finishes delivering the last batch to the respective customer and returns to the machine. Our aim is to find a schedule to minimize the C_{max}. Using the notation of Graham [2], we denote our problem as $P2 \to D, K = 1|v = 1, c = z|C_{max}$. Where "$P2 \to D, K = 1$" is used to represent that jobs are processed on either one of two identical parallel machines and then delivered to one customer, "$v = 1, c = z$" represent that one vehicle that can carry processed jobs with total sizes no more than z, and "C_{max}" is an objective function.

Chang and Lee [1] give a polynomial time algorithm with a worst-case ratio of 2 for the problem. Zhong et.al.[5] improve the upbound to 5/3. In this paper, we furthermore propose an improved algorithm with the worst-case performance ratio $\frac{14}{9} + \varepsilon$ for every $\varepsilon > 0$.

For the knapsack problem, Lawler [3] propose an $FPTAS$ with a time complexity of $O(n \log(\frac{1}{\varepsilon}) + \frac{1}{\varepsilon^4})$, where $1 - \varepsilon$ is the worst case ratio. Kellerer[4] also propose an $FPTAS$ with a time complexity of $O(n \min\{\log n, \log \frac{1}{\varepsilon}\} + \frac{1}{\varepsilon^2} \min\{n, \frac{1}{\varepsilon} \log \frac{1}{\varepsilon}\})$.

2 An improved algorithm

In the following, we first present the description of Algorithm $H2$, Procedure $\mathscr{A}$ and Procedure $\mathscr{B}$. Afterwards, we present our improved algorithm $MH2'$, which calls them when necessary. The following Algorithm $H2$ was proposed by Chang and Lee [1].

Algorithm $H2$:

Step 1. Assign jobs to batches by algorithm FFD (the jobs are ordered by nonincreasing size, and then in this order the next job is always packed into the first bin where it fits). Let the total number of resulting batches be b^{H2}.

Step 2. Calculate the sum of the processing times of the jobs in B_k and denote it P_k, for $k = 1, 2, \cdots, b^{H2}$. Reindex these batches so that $P_1 \le P_2 \le \cdots \le P_{b^{H2}}$.

Step 3. Starting with B_1, assign batches one by one to he machine (For the convenience of the analysis, we assign batch B_1 to machine M_1.) that has a smaller load before the batch is assigned (all jobs in the same batch are assigned to the same machine). Within each batch, jobs are sequenced in an arbitrary order.

Step 4. Dispatch each finished but undelivered batch whenever the vehicle becomes available. If multiple batches have been completed when the vehicle becomes available, dispatch the batch with the smallest index.

Let the schedule, makespan and objective function value obtained by algorithm $H2$ be σ^{H2}, $C(H2)$ and C^{H2}, respectively. Furthermore, let C^1 and C^2 denote the completion times of the jobs sequenced last on machine M_1 and machine M_2, respectively, in algorithm $H2$. Then $C(H2) = \max\{C^1, C^2\}$.

Remark 2.1 [1]
(1) If $C(H2)$ is determined by the first machine (denoted by $M1$), then b^{H2} is odd; If $C(H2)$ is determined by the first machine (denoted by $M2$), then b^{H2} is even.
(2) For schedule σ^{H2}, if there exists a batch B_k, $k \ge 3$, delivered immediately after being processed on M_1, then $P_k \ge 2T$; If there exists a batch B_q, $q \ge 4$, delivered

immediately after being processed on M_2, then $P_q \geq 2T$. ($T$is the transportation time including the vehicle delivers the batch to the customer and returns to the machine for each delivery.)

Remark 2.2 [1]

$C^{H2} = \max\{C(H2)+T, P_1+b^{H2}T, P-C(H2)+2T, P_2+(b^{H2}-1)T\}$.

Remark 2.3 [5]

If $C^{H2} = P_1+b^{H2}T$ ($b^{H2} \neq 1$) or $C^{H2} = P_2+(b^{H2}-1)T$ ($b^{H2} \neq 1$), then

(1) $\frac{P_1+b^{H2}T}{C^*} < \frac{2}{b^{H2}} + \frac{b^{H2}-\frac{2}{b^{H2}}}{b^*}$;

(2) $\frac{P_2+(b^{H2}-1)T}{C^*} < \frac{2}{b^{H2}-1} + \frac{b^{H2}-1-\frac{2}{b^{H2}-1}}{b^*}$.

It is shown by Chang and Lee[1] that the worst-case ratio of $H2$ is 2. Considering that there are two points which prevent the worst-case ratio of $H2$ from being better. The first one is that $H2$ assigns jobs to batches by FFD, which does not take into consideration the processing times of jobs. The total processing time of jobs in the first batch may be much larger than that in the optimal solution. Therefore, Zhong et. al.[5] use a procedure A, which applies an $FPTAS$ of the knapsack problem, enabling the vehicle to start delivering jobs earlier. The second is that $H2$ assigns jobs in one batch to a machine as a whole, regardless if the another machine may be idle at the same time. They use a procedure B, which separates the last batch into two parts. By calling the two subroutine procedures, Zhong et. al.[5] proposed their improved algorithm $MH2$ with worst-case ratio of $\frac{5}{3}$. We present the following two subroutine procedures of Procedure $\mathscr{A}$ and Procedure $\mathscr{B}$, which we will used in our algorithm in our paper.

Procedure $\mathscr{A}$:

Step 1. Construct an instance of the knapsack problem as follow, for each job J_j, $j = 1, 2.\cdots n$, we suppose its profit is p_j, size is s_j, and the knapsack capacity is z. Run any $FPTAS$ for the knapsack problem with $\varepsilon > 0$, and denote by N_1 the set of items (jobs) put into the knapsack. We also denote N_1 as a batch in the following paper when no ambiguity.

Step 2. Assign jobs $(N \backslash N_1)$ to batches by algorithm FFD. Let the total number of resulting batches be $b^{\mathscr{A}}$ (without loss of generality, we put N_1 in the last batch).

Step 3. Define $P_k^{\mathscr{A}}$ as the total processing time of the jobs in the kth batch ($k = 1, \ldots, b^{\mathscr{A}}$). Reindex these batches so that $P_1^{\mathscr{A}} \leq P_2^{\mathscr{A}} \leq \cdots \leq P_{b^{\mathscr{A}}}^{\mathscr{A}}$, and denote the kth batch as $B_k^{\mathscr{A}}$.

Step 4. Starting with $B_1^{\mathscr{A}}$, assign batches one by one to the machine(for the convenience of the analysis, we assign batch B_1 to machine M_1) that has a smaller load before the batch is assigned (all jobs in the same batch are assigned to the same machine).Within each batch, jobs are sequenced in an arbitrary order.

Step 5. Dispatch each completed but undelivered batch whenever the vehicle becomes available. If multiple batches have been completed when the vehicle becomes available, dispatch the batch with the smallest index.

Let the schedule, makespan and objective function value obtained by Procedure $\mathscr{A}$ be $\sigma^{\mathscr{A}}$, $C(\mathscr{A})$ and $C^{\mathscr{A}}$, respectively.

Procedure $\mathscr{B}$:

Let $\{J_{i_1}, J_{i_2}, \cdots, J_{i_m}\}$ be jobs in $B_{b^{H2}}$ such that $p_{i_1} \leq p_{i_2} \leq \cdots \leq p_{i_m}$, where $B_{b^{H2}}$ is the last batch of σ^{H2} obtained from Heuristic $H2$. If $C(H2) \leq \frac{3}{4}P$ or $p_{i_m} \geq \frac{2}{3}P$, then the new schedule $\sigma^{\mathscr{B}}$ is identical to σ^{H2}. Otherwise, $\sigma^{\mathscr{B}}$ is re-constructed as follows.

Step 1. If $0 < p_{i_m} \leq \frac{P}{2}$, then construct a sub-batch of $B_{b^{H2}}$ as $B^s = \{J_{i_l}, J_{i_{l+1}}, \cdots, J_{i_m}\}$, where l is the largest integer such that $\frac{P}{4} \leq \sum_{j=l}^{m} p_{i_j} \leq \frac{P}{2}$. Process and deliver batches prior to $B_{b^{H2}}$ in σ^{H2} as follows step 3 of Algorithm $H2$. Assign jobs in $B_{b^{H2}} \backslash B^s$ to the machine on which $B_{b^{H2}}$ originally was assigned, and jobs in B^s to the other machine. Deliver jobs in $B_{b^{H2}} \backslash B^s$ and B^s together as the last batch. Otherwise, go to step 2.

Step 2. If $\frac{P}{2} < p_{i_m} < \frac{2}{3}P$, let $B^s = \{J_{i_m}\}$. Assign jobs in $B_1 \cup B_2 \cup \cdots \cup B_{b^{H2}} \backslash B^s$ to the machine on which $B_{b^{H2}}$ originally was assigned (one machine starting with B_1, then $B_2, B_3, \ldots B_{b^{H2}-1}, B_{b^{H2}} \backslash B^s$) and jobs in B^s to the other machine. Deliver jobs in $B_{b^{H2}} \backslash B^s$ and B^s together as the last batch.

Let the schedule, makespan and objective function value obtained by Procedure $\mathscr{B}$ be $\sigma^{\mathscr{B}}$, $C(\mathscr{B})$ and $C^{\mathscr{B}}$, respectively. Also let P_1 and P_2 be the total processing time of jobs in the first and second batches, respectively. Based on the Algorithm $H2$, Procedure $\mathscr{A}$ and Procedure $\mathscr{B}$, we present our major algorithm $MH2'$ as follows.

Algorithm $MH2'$

Step 1. Run algorithm $H2$.

Step 2. If $C^{H2} \neq P_1 + 3T$, $C^{H2} \neq P_1 + 4T$, and $C^{H2} \neq C(H2) + T$, output C^{H2}, stop.

Step 3. If $C^{H2} = C(H2) + T$, run Procedure $\mathscr{B}$, output min $\{C^{\mathscr{B}}, C^{H2}\}$, stop.

Step 4. If $C^{H2} = P_1 + 3T$, run Procedure $\mathscr{A}$, output $\min\{C^{H2}, C^{\mathscr{A}}\}$, stop.

Step 5. If $C^{H2} = P_1 + 4T$ and $P_1 \leq \frac{1}{2}T$ or $T \leq \frac{P}{5}$, output C^{H2}, stop. Otherwise, run Procedure $\mathscr{A}$, output $\min\{C^{H2}, C^{\mathscr{A}}\}$, stop.

3 Analysis of the ratio of $MH2'$

First, we sum up the definitions and notations which will be used in the following part of the paper.

P= the total processing time of all jobs.

T= the transportation time for each delivery.

b_L^*= the number of batches if the jobs are assigned to batches by an optimal algorithm of the bin-packing problem.

P^*= the optimal value of the knapsack problem constructed in Step 1 of Procedure $\mathscr{A}$.

v_1= the departure time of jobs in the second batch in the given optimal schedule.

v_2= the total processing time of jobs in the third batch in the given optimal schedule.

B_j = the jth bin in the solution obtained by FFD in the algorithm H2, $j = 1, \ldots, b^{H2}$.

$|B_j|$ =the number of items contained in bin B_j.

E_{jl} = the lth item in B_j, $j = 1, \ldots, 4, l = 1, \ldots, |B_j|$.

For our algorithm and procedures, we introduce the following notations. Show in table 1.

Before proceeding, we first introduce several lemmas on the procedures and our algorithm $MH2'$.

Lemma 3.1 *If $b^* \leqslant 3$, $\sum_{i=1}^{k} s_i \leq \frac{7z}{3}$ ($k \leq n$), then the k items can be assigned to three bins by algorithm FFD.*

Proof : Suppose that there are at least four bins to pack the k items by FFD algorithm of Bin-packing problem. Let S'_j ($j = 1, 2, 3$) be the jth bin's total items size when E_{41} is packed to the fourth bin, then $S'_1 + E_{41} > z, S'_2 + E_{41} > z, S'_3 + E_{41} > z, S'_1 + S'_2 + S'_3 + E_{41} \leq \frac{7z}{3}$. We have $E_{41} > \frac{1}{3}z$, and there are at most two items in each of the first three bins. Thus there are at least four bins to optimally pack the $k$items, contradicting to $b^* = 3$.

Corollary 3.1 *For $N_1 \subset N$, $S - S_{N_1} \leq \frac{7z}{3}$ and $N \backslash N_1$ can be packed into $\overline{b} - 1$ bins, we have*
(1) If $\overline{b} = 4$, then $b^* \leq 3$;
(2) If $\overline{b} > 4$, then $b^* > 3$.

Lemma 3.2 *For an instance I of the bin-packing problem, let $OPT(I)$, $FFD(I)$, $FF(I)$ be the number of used bins in an optimal solution, the solutions yielded by FFD and FF, respectively, we have*
(1)$FF(I) \leq \frac{7}{4}OPT(I)$;[6]
(2)$FFD(I) \leq \frac{11}{9}OPT(I) + \frac{7}{9}$.[7]

Lemma 3.3 *For $b^{H2} \neq 3$ or 4 and $C^{H2} \neq C(H2)+T$, if $C^{H2} = P_1 + b^{H2}T$ or $C^{H2} = P_2 + (b^{H2} - 1)T$, then $\frac{C^{H2}}{C^*} \leq \frac{14}{9}$.*

Proof : Recall that $b^{H2} \leq \frac{3}{2}b^*_L \leq \frac{3}{2}b^*$ and $b^{H2} \leq \frac{11}{9}b^*_L + \frac{7}{9} \leq \frac{11}{9}b^* + \frac{7}{9}$.
If $b^{H2} = 1$, then $C^{H2} = C(H2) + T$, contradicting the assumption $C^{H2} \neq C(H2) + T$.
If $b^{H2} = 2$, $b^* \geq 2$, we only consider the case that $C^{H2} = P_1 + b^{H2}T$ due to $C^{H2} = P_2 + (b^{H2} - 1)T = C(H2) + T$. We have $\frac{C^{H2}}{C^*} \leq \frac{3}{2} \leq \frac{14}{9}$ from Remark 2.3.
If $b^{H2} = 5$ *or* 6 *or* 7, we have $b^* \geq b^{H2} - 1$. We also have $\frac{C^{H2}}{C^*} \leq \frac{14}{9}$ from Remark 2.3.
If $b^{H2} = 8$ *or* 9, we have $b^* \geq b^{H2} - 2$, We also have $\frac{C^{H2}}{C^*} \leq \frac{14}{9}$ from Remark 2.3.
If $b^{H2} \geq 10$, $\frac{P_1+b^{H2}T}{C^*} < \frac{2}{b^{H2}} + \frac{11}{9}\frac{b^{H2}-\frac{2}{b^{H2}}}{b^{H2}-1}$ and $\frac{P_2+(b^{H2}-1)T}{C^*} < \frac{2}{b^{H2}-1} + \frac{11}{9}\frac{b^{H2}-1-\frac{2}{b^{H2}-1}}{b^{H2}-1}$.
Define $f(P_1) = \frac{2}{P_1} + \frac{11}{9}\frac{P_1-\frac{2}{P_1}}{P_1-1} = \frac{1}{9}(\frac{40}{P_1} - \frac{11}{P_1-1} + 11)$. Then $f'(P_1) = \frac{1}{9}(-\frac{40}{P_1^2} + \frac{11}{(P_1-1)^2})$. It can be easily verified that $f(10) = 1.53 < \frac{14}{9}$, $f'(P_1) < 0$ for $P_1 \geq 10$. Thus $\frac{P_1+b^{H2}T}{C^*} < \frac{14}{9}$ and $\frac{P_2+(b^{H2}-1)T}{C^*} < \frac{14}{9}$.

Lemma 3.4 *If $C^{H2} = P - C(H2) + 2T$ ($b^{H2} \geq 3$), then $\frac{C^{H2}}{C^*} \leq \frac{14}{9}$.*

Proof : If $b^{H2} = 3$, then $b^* \geq 2$. By Remark 2.3, we have $\frac{C^{H2}}{C^*} < \frac{3}{2} < \frac{14}{9}$.
If $b^{H2} \geq 4$, from $P - C(H2) + 2T \geq P_1 + 4T \geq 4T$, we have $P \geq 2T$.
As $C(H2) \geq \frac{P}{2}$ and $C(*) \geq \frac{P}{2}$, we get

$$\frac{C^{H2}}{C^*} \leq \frac{P-C(H2)+2T}{C(*)+T} \leq \frac{P-\frac{P}{2}+2T}{\frac{P}{2}+T} = 1 + \frac{T}{\frac{P}{2}+T} \leq \frac{3}{2} < \frac{14}{9}.$$

Lemma 3.5 *If $C^{H2} = C(H2) + T$, $C(H2) \leq \frac{3}{4}P$ or $p_{i_m} \geq \frac{2}{3}P$, then $\frac{C^{H2}}{C^*} \leq \frac{3}{2}$.*

Proof : Note that $\frac{C^{H2}}{C^*} \leq \frac{C(H2)+T}{C(*)+T}$.
If $C(H2) \leq \frac{3}{4}P$, then

$$\frac{C^{H2}}{C^*} \leq \frac{C(H2)+T}{C(*)+T} \leq \frac{\frac{3}{4}P+T}{\frac{P}{2}+T} = \frac{3}{2} - \frac{T/2}{\frac{P}{2}+T} \leq \frac{3}{2}.$$

If $p_{i_m} \geq \frac{2}{3}P$, then $C(H2) \geq \frac{2}{3}P$, therefore

$$\frac{C^{H2}}{C^*} \leq \frac{C(H2)+T}{C(*)+T} \leq \frac{P+T}{\frac{2}{3}P+T} = 1 + \frac{\frac{1}{3}P}{\frac{2}{3}P+T} \leq 1 + \frac{\frac{1}{3}P}{\frac{2}{3}P} = \frac{3}{2}.$$

Lemma 3.6 *(1) If $\sigma^{\mathscr{B}}$ is re-constructed by Procedure $\mathscr{B}$, then $C(\mathscr{B}) \leq \frac{3}{4}P$;*
(2) If $\sigma^{\mathscr{B}}$ is re-constructed in Step 1 of Procedure $\mathscr{B}$, then $C^{\mathscr{B}} \in \{P_1 + b^{H2}T, P_2 + (b^{H2} - 1)T, C(\mathscr{B}) + T\}$;
(3) If $\sigma^{\mathscr{B}}$ is re-constructed in Step 2 of Procedure $\mathscr{B}$, then $C^{\mathscr{B}} \in \{P_1 + b^{H2}T, C(\mathscr{B}) + T, P - P_{b^{H2}} + 2T\}$.

Proof : (1) Note that $\sigma^{\mathscr{B}}$ is re-constructed by Procedure $\mathscr{B}$ only when $C(H2) > \frac{3}{4}P$ and $p_{i_m} < \frac{2}{3}P$.
If $0 < p_{i_m} < \frac{1}{2}P$, then

$$B^s = \{J_{i_l} J_{i_{l+1}} \cdots J_{i_m}\} \text{ and } \frac{P}{4} \leq \sum_{j=l}^{m} p_{i_j} \leq \frac{P}{2}, \text{ so}$$

$$C(\mathscr{B}) = \max\{C(H2) - \sum_{j=l}^{m} p_{i_j}, P - C(H2) + \sum_{j=l}^{m} p_{i_j}\} \leq \frac{3P}{4}.$$

If $\frac{1}{2}P < p_{i_m} < \frac{2}{3}P$, then

$$B^s = \{J_{i_m}\}, \text{ and } C(\mathscr{B}) = \max\{p_{i_m}, P - p_{i_m}\} = p_{i_m} \leq \frac{3P}{4}.$$

(2) If $b^{H2} = 1$, we have $C^{\mathscr{B}} = C(\mathscr{B}) + T$.
If $b^{H2} = 2$, we consider the time when the vehicle finishes delivering B_1 and returns to the machines. If it is bigger than $C(\mathscr{B})$, then $C^{\mathscr{B}} = P_1 + 2T$. Otherwise, $C^{\mathscr{B}} = C(\mathscr{B}) + T$.
If $b^{H2} \geq 3$, we first suppose that $C(H2)$ is determined by $M1$. Consider the time when the vehicle finishes

Table 1: Notations of algorithm $MH2'$.

Algorithm (Procedure)	$H2$	$\mathscr{A}$	$\mathscr{B}$	Optimal schedule
Schedule	σ^{H2}	$\sigma^{\mathscr{A}}$	$\sigma^{\mathscr{B}}$	$-$
The number of batches	b^{H2}	$b^{\mathscr{A}}$	b^{H2}	b^*
The objective function value	C^{H2}	$C^{\mathscr{A}}$	$C^{\mathscr{B}}$	C^*
The makespan	$C(H2)$	$C(\mathscr{A})$	$C(\mathscr{B})$	$C(*)$
The departuretime of jobs in the first batch	P_1	$P_1^{\mathscr{A}}$	$-$	u
The total processing time of jobs in the second batch	P_2	$P_2^{\mathscr{A}}$	$-$	v

delivering the second last batch $B_{b^{H2}-1}$ and returns to the machines, which is denoted by F. Note that F must be one of the following four values, $P_1 + (b^{H2} - 1)T$, $P - C(H2) + T$, $P_2 + (b^{H2} - 2)T$, or $C(H2) - P_{b^{H2}} + 2T$. If $F = P_1 + (b^{H2} - 1)T$, it is obvious that $C^{\mathscr{B}} = P_1 + b^{H2}T$ or $C^{\mathscr{B}} = C(\mathscr{B}) + T$. For the remaining cases, we will prove that $F < \frac{P}{2} \leq C(\mathscr{B})$, and thus $C^{\mathscr{B}} = C(\mathscr{B}) + T$.

If $F = P - C(H2) + T$, then $B_{b^{H2}-1}$ is delivered immediately after being processed. From remark 2.3 and $C(H2) > \frac{3P}{4}$, it follows that $2T \leq P_{b^{H2}-1} < P - C(H2) < \frac{P}{4}$ and $F = P - C(H2) + T < \frac{P}{4} + \frac{P}{8} < \frac{P}{2}$.

If $F = P_2 + (b^{H2} - 2)T$, then the second batch is delivered immediately after being processed. Hence, $P_2 \geq P_1 + T > T$. Combining that $C(H2) > \frac{3P}{4}$ and $P_1 \leq P_2 \leq \cdots \leq P_{b^{H2}}$, we have

$P_2 \leq \frac{P - C(H2)}{\frac{b^{H2}-1}{2}} < \frac{\frac{P}{4}}{\frac{b^{H2}-1}{2}} = \frac{P}{2(b^{H2}-1)}$ and $F = P_2 + (b^{H2} - 2)T < P_2 + (b^{H2} - 2)P_2 = (b^{H2} - 1)P_2 < \frac{P}{2}$.

If $F = C(H2) - P_{b^{H2}} + 2T$, then $B_{b^{H2}-2}$ is delivered immediately after being processed. From remark 2.3 and $C(H2) > \frac{3P}{4}$, it follows that

$2T \leq P_{b^{H2}-2} \leq P_{b^{H2}-1} < P - C(H2) < \frac{P}{4}$ and $F = C(H2) - P_{b^{H2}} + 2T < \frac{P}{4} + \frac{P}{4} = \frac{P}{2}$.

If $C(H2)$ is determined by $M2$, the result can be proved similarly.

(3) If $\sigma^{\mathscr{B}}$ is re-constructed in Step 2 of Procedure $\mathscr{B}$, it is easy to show that $F = P_1 + (b^{H2} - 1)T$ or $F = P - P_{b^{H2}} + T$. Hence

$$C^{\mathscr{B}} \in \{P_1 + b^{H2}T, C(\mathscr{B}) + T, P - P_{b^{H2}} + 2T\}.$$

Lemma 3.7 *If $C^{H2} = C(H2) + T$, then $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$.*

Proof : If $\sigma^{\mathscr{B}}$ is identical to σ^{H2},then $C(H2) \leq \frac{3}{4}P$ or $p_{i_m} \geq \frac{2}{3}P$, so we have $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$ by Lemma 3.5.

When $\sigma^{\mathscr{B}}$ is re-constructed in Step 1 of Procedure $\mathscr{B}$, then from Lemma 3.6 we have

$$C^{\mathscr{B}} \in \{P_1 + b^{H2}T, P_2 + (b^{H2} - 1)T, C(\mathscr{B}) + T\}.$$

If $C^{\mathscr{B}} = C(\mathscr{B}) + T$, then similarly as in Lemma 3.5 implies that $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$ due to $C(\mathscr{B}) \leq \frac{3P}{4}$.

If $C^{\mathscr{B}} = P_2 + (b^{H2} - 1)T$, then from remark 2.3, we have $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$.

If $C^{\mathscr{B}} = P_1 + b^{H2}T$, when $b^{H2} \neq 3, 4$ and $b^{H2} \geq 2$, we have already proved that $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$ in Lemma 3.3.

Next we show that $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{3}{2}$ for $C^{\mathscr{B}} = P_1 + b^{H2}T$ and $b^{H2} = 3$ *or* 4.

If $C^{\mathscr{B}} = P_1 + 3T$, then $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{P_1 + 3T}{\frac{P}{2} + T}$. From $C^{H2} = C(H2) + T$, we have $P_3 \geq 2T$ form remark 2.1. Combining $P - 2P_1 \geq P_3 \geq 2T$, we get $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{P_1 + 3T}{\frac{P}{2} + T} < \frac{3}{2}$.

If $C^{\mathscr{B}} = P_1 + 4T$. Note that there are two batches with the total load $P - C(H2) < \frac{P}{4}$ on $M1$, thus $P_1 < \frac{P}{8}$, and then $\frac{C^{\mathscr{B}}}{C^*} \leq \frac{\frac{1}{4}(\frac{P}{2} + T)}{\frac{P}{2} + T} + \frac{\frac{15}{4}T}{u + 3T} < \frac{3}{2}$.

When $\sigma^{\mathscr{B}}$ is re-constructed in Step 2 of Procedure $\mathscr{B}$, then from Lemma 3.6 we have

$$C^{\mathscr{B}} \in \{P_1 + b^{H2}T, C(\mathscr{B}) + T, P - P_{b^{H2}} + 2T\}.$$

If $C^{\mathscr{B}} \in \{P_1 + b^{H2}T, C(\mathscr{B}) + T\}$, we have proved that $\frac{C^{\mathscr{B}}}{C^*} < \frac{3}{2}$ in the front half part of the theorem. Next, we assume that $C^{\mathscr{B}} = P - P_{b^{H2}} + 2T$ ($b^{H2} \geq 3$).

As $P_{b^{H2}} \geq C(\mathscr{B})$, we have

$$P - C(\mathscr{B}) + T \geq P - P_{b^{H2}} + T > P_1 + (b^{H2} - 1)T,$$
$$\text{and } P - C(\mathscr{B}) > P_1 + (b^{H2} - 2)T.$$

Combining that $\frac{P}{2} \geq P - C(\mathscr{B})$, we get $\frac{P}{2} > T$. Thus

$$\frac{C^{\mathscr{B}}}{C^*} \leq \frac{P - C(\mathscr{B}) + 2T}{\frac{P}{2} + T} \leq \frac{\frac{P}{2} + 2T}{\frac{P}{2} + T} = 1 + \frac{T}{\frac{P}{2} + T} < \frac{3}{2}.$$

Lemma 3.8 *If $b^* = 2$ and $b^{\mathscr{A}} = 3$, then $P_1^{\mathscr{A}} \leq \frac{\varepsilon P}{2} + (1 - \varepsilon)u$.*

Proof : From the property of $FPTAS$ of the knapsack problem, we know that $P_3^{\mathscr{A}} \geq (1 - \varepsilon)P^* \geq (1 - \varepsilon)v$. As $P \leq 2u + v$, we have $v \geq P - 2u$.

Complying with ${P_1}^{\mathscr{A}} \leq {P_2}^{\mathscr{A}} \leq {P_3}^{\mathscr{A}}$, we get

$$P_1^{\mathscr{A}} \leq \frac{P - {P_3}^{\mathscr{A}}}{2} \leq \frac{P - (1 - \varepsilon)(P - 2u)}{2} = \frac{\varepsilon P}{2} + (1 - \varepsilon)u.$$

Lemma 3.9 *If $b^* = 2$ and $C^{\mathscr{A}} \neq C(\mathscr{A}) + T$, then $\frac{C^{\mathscr{A}}}{C^*} < \frac{14}{9} + \varepsilon$.*

Proof : From Lemma 3.2 (1) , we have $b^{\mathscr{A}} \leq 3$.

If $b^{\mathscr{A}} = 2$, we get $C^{\mathscr{A}} = P_1^{\mathscr{A}} + 2T = P - C(\mathscr{A}) + 2T$.

From Lemma 3.3, we obtain that $\frac{C^{\mathscr{A}}}{C^*} < \frac{3}{2}$.

If $b^{\mathscr{A}} = 3$, since $C^{\mathscr{A}} \neq C(\mathscr{A}) + T$, then

$$C^{\mathscr{A}} = P_1^{\mathscr{A}} + 3T \text{ or } P - C(\mathscr{A}) + 2T = P_2^{\mathscr{A}} + 2T.$$

For the case that $P-C(\mathscr{A})+2T=P_2^{\mathscr{A}}+2T$, it is easy to get that $\frac{C^{\mathscr{A}}}{C^*}<\frac{3}{2}$ from remark 2.3. The following part, we will prove that $\frac{C^{\mathscr{A}}}{C^*}=\frac{P_1^{\mathscr{A}}+3T}{C^*}<\frac{3}{2}+\varepsilon$ when $C^{\mathscr{A}}=P_1^{\mathscr{A}}+3T$. From Lemma 3.8, we know that $P_1^{\mathscr{A}}\leq\frac{\varepsilon P}{2}+(1-\varepsilon)u$. As $C^*\geq\max\{C(*)+T,u+2T\}$, we consider the following two cases.

Case 1. If $u+2T>C(*)+T$. Then

$u+T>C(*)\geq\frac{P}{2}$, thus $\frac{P}{2}-u\leq T$. Hence

$$\frac{C^{\mathscr{A}}}{C^*}\leq\frac{P_1^{\mathscr{A}}+3T}{u+2T}\leq\frac{\frac{\varepsilon P}{2}-\varepsilon u+u+3T}{u+2T}\leq 1+\frac{\varepsilon(\frac{P}{2}-u)+T}{u+2T}$$

$$=1+\frac{\varepsilon T+T}{u+2T}\leq\frac{3}{2}+\varepsilon.$$

Case 2. If $u+2T<C(*)+T$. Then

$u+T<C(*)<P$, thus $u<C(*)-T$.Hence

$$\begin{aligned}\frac{C^{\mathscr{A}}}{C^*} &\leq \frac{P_1^{\mathscr{A}}+3T}{C(*)+T}\leq\frac{\frac{\varepsilon P}{2}+(1-\varepsilon)u+3T}{C(*)+T}\\ &< \frac{\varepsilon C(*)+(1-\varepsilon)[C(*)-T]+3T}{C(*)+T}\\ &= \frac{C(*)+2T+\varepsilon T}{C(*)+T}\\ &= 1+\frac{T+\varepsilon T}{C(*)+T}\\ &\leq \frac{3}{2}+\varepsilon.\end{aligned}$$

Lemma 3.10 *If $C^{H2}=P_1+3T$, then $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$.*

Proof : If $C^{H2}=P_1+3T$, then $b^*=2$ or $b^*=3$ by Lemma 3.2 (2).

If $b^*=2$, we have $\frac{C^{MH2'}}{C^*}<\frac{3}{2}+\varepsilon$ from Lemma 3.9.

If$b^*=3$, we also have $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$ from remark 2.3.

Lemma 3.11 *If $C^{H2}=P_1+4T$, then $\frac{C^{MH2'}}{C^*}<\frac{14}{9}+\varepsilon$.*

Proof : By Lemma 3.2 (2), we have $b^*=3$ or $b^*=4$. If $b^*=4$, it is easy to get $\frac{C^{MH2'}}{C^*}<\frac{3}{2}+\varepsilon<\frac{14}{9}+\varepsilon$ from remark 2.3. If $b^*=3$, we consider the following two cases.

Case 1. $P_1\leq\frac{1}{2}T$ or $T\leq\frac{P}{5}$.

As $C^*\geq\max\{C(*)+T,u+3T\}$, thus

$\frac{C^{H2}}{C^*}\leq\frac{P_1+4T}{u+3T}<\frac{3}{2}$ or $\frac{C^{H2}}{C^*}\leq\frac{P_1+4T}{C(M)^*+T}\leq\frac{\frac{1}{4}P+4T}{\frac{P}{2}+T}<\frac{3}{2}$.

Case 2. $P_1>\frac{1}{2}T$ and $T>\frac{P}{5}$.

If $S_{N_1}\geq\frac{2z}{3}$, note that $b^*=3$, we have $S-S_{N_1}\leq\frac{7z}{3}$. Thus $b^{\mathscr{A}}\leq 4$ from Lemma 3.1.

If $\frac{z}{2}<S_{N_1}<\frac{2z}{3}$, then there are at most 6 items in $N\backslash N_1$ and each item size is larger than $\frac{z}{3}$, therefore the items in $N\backslash N_1$ can be assigned three bins by algorithm FFD (if not $b^*\neq 3$), therefore $b^{\mathscr{A}}\leq 4$.

If $S_{N_1}\leq\frac{z}{2}$, then there are at most 3 items in $N\backslash N_1$ and each item size is larger than $\frac{z}{2}$, therefore $b^{\mathscr{A}}\leq 4$.

If $b^*=3, b^{\mathscr{A}}=3$, it is easy to obtain $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$ from remark 2.3. Next, we only need to consider the case of $b^*=3$ and $b^{\mathscr{A}}=4$.

If $C^{\mathscr{A}}=P_2^{\mathscr{A}}+3T$, then $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$ from remark 2.3.

If $C^{\mathscr{A}}=C(\mathscr{A})+T$, then $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$ from Lemma 3.7.

If $C^{\mathscr{A}}C=P-C(\mathscr{A})+2T$, then $\frac{C^{MH2'}}{C^*}<\frac{3}{2}$ from Lemma 3.4.

If $C^{\mathscr{A}}=P_1^{\mathscr{A}}+4T$. Noting that $P\leq 2v_1+v_2$, we have $v_2\geq P-2v_1$. As

$P_4^{\mathscr{A}}\geq(1-\varepsilon)P^*\geq(1-\varepsilon)v_2=(1-\varepsilon)(P-2v_1)$, and

$P_1^{\mathscr{A}}\leq\frac{P-P_4^{\mathscr{A}}}{3}\leq\frac{P-(1-\varepsilon)(P-2v_1)}{3}=\frac{\varepsilon P+2v_1(1-\varepsilon)}{3}$. Thus

$$\begin{aligned}\frac{C^{MH2'}}{C^*} &= \frac{P_1{}^{\mathscr{A}}+4T}{C^*}\\ &\leq \frac{\frac{\varepsilon P+2v_1(1-\varepsilon)}{3}+4T}{C^*}\\ &= \frac{\frac{1-\varepsilon}{3}(2v_1+4T)+\frac{2\varepsilon}{3}(\frac{P}{2}+T)+(\frac{8}{3}+\frac{2}{3}\varepsilon)T}{C^*}\\ &\leq \frac{\frac{1-\varepsilon}{3}(2v_1+4T)}{v_1+2T}+\frac{\frac{2\varepsilon}{3}(\frac{P}{2}+T)}{\frac{P}{2}+T}+\frac{\frac{8+2\varepsilon}{3}T}{3T}\\ &= \frac{2(1-\varepsilon)}{3}+\frac{2\varepsilon}{3}+(\frac{8+2\varepsilon}{9})\\ &= \frac{14}{9}+\frac{2\varepsilon}{9}\\ &= \frac{14}{9}+\varepsilon\ \ (when\ \ \varepsilon\to 0).\end{aligned}$$

As a direct conclusion of the above Lemmas 3.3, 3.4, 3.7, 3.10 and 3.11, we obtain the following main result.

Theorem 3.1 $\frac{C_{MH2'}}{C^*}\leq\frac{14}{9}+\varepsilon$*, where $C_{MH2'}$ is the objective function value obtained from algorithm $MH2'$.*

4 Conclusions

In this paper, we present an algorithm with the worst-case performance ratio of $\frac{14}{9}+\varepsilon$, which improves the upper bound of $\frac{5}{3}$ obtained by a algorithm gained by Zhong et.al.[5]. In order to get the ratio of $\frac{14}{9}+\varepsilon$, two new procedures of the improved algorithm used to handle difficult cases such as $b^{H2}=3$ and $b^{H2}=4$. Obtaining the best possible algorithm or improving the lower bound is an interesting an challenging task.

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References

[1] Chang, Y.C., and Lee, C.Y., Machine scheduling with job delivery coordination, E. J. O. R., 2004, 158, (2), pp. 470-487.

[2] Graham, R.L., Lawler, E.L., Lenstra, J.K., et al. Optimization and approximation in deterministic sequencing and scheduling, Annals of Discrete Mathematics, 1979, 5, pp. 287-326.

[3] Lawler, E.L., Fast approximation algorithms for knapsack problems, Mathematics of Operations Research, 1979, 4, (4), pp. 339-356.

[4] Kellerer, H., Pferschy, U., A New Fully Polynomial Approximation Scheme for the Knapsack Problem, Lecture Notes in Computer Science, 1998, 1444, pp. 123-134.

[5] Zhong, W., Dosa, G., Tan, Z.Y., On the machine scheduling problem with job delivery coordination, European Journal of Operational Research, 2007, 182, (3), pp. 1057-1072.

[6] Simchi, L.D., New worst-case results for the bin-packing problem, Naval Research Logistics, 1994, 41, (4), pp. 579-585.

[7] Dósa, G., Li, R.H., Han, X., Tuza, Z., Tight absolute bound for First Fit Decreasing bin-packing: $FFD(L) \leqslant 11/9OPT(L) + 6/9$, Theoretical Computer Science, 2013, 510, pp. 13-61.

[8] He, Y., Zhong, W., Gu, H., Improved Algorithms for Two Single Machine Scheduling Problems, Theoretical Computer Science, 2006, 363, (3), pp. 257-265.