The Ninth International Symposium on Operations Research and Its Applications (ISORA'10)
Chengdu-Jiuzhaigou, China, August 19–23, 2010
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Determining Storage Locations and Capacities for Emergency Response*

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Abstract In this paper, we consider the problem of emergency response resource storage locations and capacities. Resources are important for disaster relief operations in coping with natural and manmade emergencies. In order to enhance the ability of response to disasters, different levels of reserve system need to be set up for holding relief resources. These emergency storages may have different capacities and construction costs. In this paper, we formulate a model to determine the storage locations and their capacities at minimum total construction cost. Then, an approximation algorithm is presented by introducing LP-rounding technique. Finally, the proofs of correctness and approximation ratio are shown.

Keywords Location; Resource Reserve System; LP-rounding; Approximation Algorithm; Emergency Response

1 Introduction

A disaster is defined by the World Health Organization as any occurrence that causes damage, destruction, ecological disruption, loss of human life, human suffering, deterioration of health and health services on a scale sufficient to warrant an extraordinary response from outside the affected community or area. Earthquakes, hurricanes, tornadoes, volcanic eruptions, fire, floods, blizzard, drought, terrorism, chemical spills, nuclear accidents are included among the causes of disasters, and all have significant devastating effects in terms of human injuries and property damage.

In order to enhance the capacity of response to disasters, the Chinese central government is planning a national reserve system of disaster relief commodities such as tents, food, drinking water, medicines and equipment, etc. Therefore, the government administration needs to determine where these inventories locate and and what is the capacity of each storage. Generally, there can be several levels of capacities for a storage to be

*This work was supported in part by National Natural Science Foundation of China under Grand No. 90924008, and the President Fund of Graduate University of Chinese Academy of Sciences.
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determined. It is also assumed that there exist a set of potential locations to set up a inventory. These places should always be easy to access by current transport infrastructure, in order to transport resources while emergency events happen. Obviously, different size of storage for emergency resources needs different construction cost. On the other hand, the storage capacity must meet the demand so as to provide sufficient disaster relief operation resources in response to emergency situations. This process can be divided into two stages. Firstly, according to the potential emergency disasters, affected areas can be pointed out for a special disaster. Secondly, the requirement at a potential affected area can be estimated by the population in this area.

Beginning with Cooper [3] in 1963, facility location-allocation (FLA) provides a valuable method in deciding where to place facilities coupled with determining how to assign demand to the located facilities in order to utilize resources effectively. Logendran and Terrell [7] firstly introduced the stochastic uncapacitated FLA model in which customers' demands was assumed to be random variables. Next, by introducing fuzzy theory into FLA problem, the capacitated FLA problem with fuzzy demands of customers as the expected cost minimization model, K-cost minimization model and possibility maximization model were formulated (Zhou and Liu [14]). Wen and Iwamura [12] considered the FLA problem under random fuzzy environment using (K,L)-cost minimization model under the Hurwicz criterion.

Nozick and Turnquist [8] present a model to choose distribute center (DC) locations and allocations in a multi-product system. Their model can be used, for example, to decide which products to stock at a central plant, which to stock at regional DCs, and which not to stock at all (i.e., produce in a make-to-order fashion). They propose an iterative approach that alternately solves a uncapacitated fixed-charge location problem and a stocking problem (for a fixed set of DC locations). Stocking decision is an important issue in the inventory system. Kukreja et al.[6] develop stocking policies for low usage items in a multi-location inventory system with all locations using a continuous review, one-for-one ordering policy.

Beamon and Kotleba [2] developed an inventory model for a pre-positioned warehouse responding to a complex humanitarian emergency. Complex humanitarian emergencies are unique due to their unpredictable demand patterns and long durations. Their model investigated a single ąřitemąś (which may be interpreted as a single type of relief kit or a single set of items), developed order quantities that were independent of vehicle or container sizes, and assumed a continuous demand approximation.

Emergency logistics has also emerged as a worldwide-noticeable theme. Sheu [10] presented four main challenges that emergency logistics management can be characterized. Also as a sponsor, Sheu edited a special issue of Transportation Research Part E, in which six papers on emergency logistics were included. These papers concentrated on addressing the issue of relief distribution to affected areas. In some of the studies, the issue of evacuating affected people was also considered. When dealing with uncertainty, Potvin et al.[9] list a number of reactive dynamic strategies for vehicle routing and scheduling problems.

One of the earliest studies conducted on location of emergency service facilities is by Toregas et al.[11] modeling the problem as a set covering problem and using a linear programming as the solution method. Consignment of goods is typically examined in the literature as a multi-commodity network flow problem, with a multi-period and/or multi-modal setting. Haghani and Oh [5] formulated a multi-commodity, multi-modal network flow model with time windows for disaster response. Two heuristic algorithms are proposed. The flow of goods over an urban transportation network is modeled as a multi-commodity, multi-modal network flow problem by Barbarosoglu et al. [1]. A two-stage stochastic programming framework is formed as the solution approach. Another study on the topic, conducted by Fiedrich et al.[4], model the problem similar to a machine scheduling problem proposing two heuristics, Simulated Annealing and Tabu Search. Yi and Ozdamar [13] consider a dynamic and fuzzy logistics coordination model for conducting disaster response activities. The model is illustrated on an earthquake data set from Istanbul. Also, Barbarosoglu et al. [1] proposed that their model could be used effectively within a decision-aid tool by public and non-public response agencies that are obscured by the variability of impact estimations under large number of different earthquake scenarios.

Mathematical modeling of storage location and inventory management in emergency relief efforts has received little attention in the literature. In order to ensure continuous capacity in long-term responses, government organizers often institute arrangements for the storage of relief items in warehouses located near the response location. This research focuses on developing an warehouses location and inventory level determination management policy to improve the effectiveness and efficiency of emergency relief during long-term humanitarian responses. In this paper, we consider a special kind of resource, such as tent, vaccine, etc.

The remainder of this paper is organized as follows. In Section 2, we formulate a storage location and capacity determination problem and specify that this model is NP-hard. In Section 3, an approximation algorithm for solving this problem is presented and the approximation ratio is analyzed. Finally, we conclude the paper with a summary and directions for future research.

2 Model Description for Location and capacity Determination Problem

In this paper, we consider a resource reserve system with single type of commodity, such as tents. Now, several warehouses for holding the commodity need be built in order to deal with disasters in the special area. Decision maker needs to determine location and size of these inventories.

Let P represent the set of potential locations. Suppose there are L levels of the storage sizes. And C_l and c_l are the capacity and setting up cost of storage at level l respectively.

Parameters

J	:	Set of affected districts
C_l	:	capacity of storage at level l
c_l	:	cost of setting up a storage at level l
d_j	:	the potential demand for some commodity in affected district i.
T	:	the expected deadline for the commodity, which means the commodity should arrive in T after a disaster
t_{ij}	:	travel time from i to j

$$a_{ij} = \begin{cases} 1, & \text{if} \quad t_{ij} \le T, \\ 0, & \text{if} \quad t_{ij} > T, \end{cases}$$

Our optimization model will determine the minimum total cost at which a set of storages can be constructed, such that the demand of each affected district can be satisfied. Up to these parameters, the following decision variables need to be determined.

Decision variables

$$x_i^l = \begin{cases} 1, & \text{set up a storage in level } l \text{ at location } i, \\ 0, & \text{ortherwise,} \end{cases}$$

Then, The model for storage location and level determination problem (SLLD) can be stated as follows:

$$\textbf{min} \quad z = \sum_{i \in P} \sum_{l=0}^{l=L} c^l x_i^l \tag{1}$$

subject to

$$\sum_{l=0}^{l=L} x_i^l = 1, \quad \forall i \in P \tag{2}$$

$$\sum_{i \in P} \sum_{l=0}^{l=L} a_{ij} C^l x_i^l \ge d_j, \quad \forall j \in J \tag{3}$$

$$x_i^l \in \{0,1\} \quad \forall i \in P, l = 1,2,\ldots,L \tag{4}$$

The objective function (1) serves to minimize the total cost of constructing all storages. Constraints (2) specify only one type level can be chosen at each potential location. Constraints (3) guarantee that the required amount of demand is satisfied. Finally, (4) states that x_i^l-variables are binary.

This problem SLLD is NP-hard, which can be proved by showing set cover problem is a special case of this problem.

3 Approximation Algorithm Design and Analysis

In this section, we will present an approximation algorithm by introducing LP-rounding technique for SLLD. Then, the theoretical performance of this algorithm is analyzed.

Firstly, by relaxing integer constraints of SLLD, we obtain the LP-relaxation of SLLD as follows:

$$\textbf{min} \quad z = \sum_{i \in P} \sum_{l=0}^{l=L} c^l x_i^l \tag{5}$$

subject to

$$\sum_{l=0}^{l=L} x_i^l = 1, \quad \forall i \in P \tag{6}$$

$$\sum_{i \in P} \sum_{l=0}^{l=L} a_{ij} C^l x_i^l \geq d_j, \quad \forall j \in J \tag{7}$$

$$0 \leq x_i^l \leq 1, \quad \forall i \in P, l = 1, 2, \ldots, L \tag{8}$$

$$\tag{9}$$

The LP-relaxation of SLLD can be solved efficiently since it is a linear programming. Now, we propose an LP-rounding algorithm for the SLLD. This algorithm contains three stage: solving the LP-relaxation of SLLD, solution rounding and solution adjustment.

The following process contains steps of our LP-rounding Algorithm (LPrA).

Algorithm LPrA LP-ROUNDING ALGORITHM FOR SLLD

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1. solve the LP-relaxation of SLLD, and get fractional optimal solution x_i^{l*}.
2. **for** each $i \in P$
3. $\quad s_i^* = \sum_{l=0}^{l=L} C^l x_i^{l*}$
4. $\quad l_i^* = min\{l | C^l \geq s_i^*, l = 0, 1, 2, \ldots, L\}$
5. $\quad x_i^l = \begin{cases} 1, & l = l_i^*, \\ 0, & \text{ortherwise}, \end{cases}$
6. **endfor**
7. Introducing temporary variable $y_i^l = x_i^l$
8. **for** each $i \in P$
9. $\quad$ **if** $x_i^l = 1$ and $l \neq 0$
10. $\quad\quad$ Let $y_i^l = 0$ and $y_i^{l-1} = 1$, if for any $j \in J$, $\sum_{i \in P} \sum_{l=0}^{l=L} a_{ij} C^l y_i^l \geq d_j$ still hold, then set $x_i^l = 0$ and $x_i^{l-1} = 1$.
11. $\quad$ **endif**
12. **endfor**
13. Output the integer solution x_i^l.

In LPrA, LP-relaxation of SLLD is solved in step 1 and obtain the fractional optimal solution x_i^{l*}. The rounding process is from step 2 to step 6, after which we get the integer solution. In order to improve the performance of our algorithm, the adjustment process from step 8 to 11 is designed.

Before the proof of correctness and performance for LPrA, let $z^*(SLLD)$ and $z^*(LP-SLLD)$ be the optimal value for SLLD and LP-relaxation of SLLD. Obviously, we have

$$z^*(SLLD) \geq z^*(LP-SLLD).$$

Now, let's define:

$$c_0 = |P|c^1 \tag{10}$$

$$r = max\{\frac{c^{l+1}}{c^l}|l = 1,2,\ldots,L-1\} \tag{11}$$

Since linear programming can be solved in polynomial time, the LPrA is obvious a polynomial time algorithm. For the performance of LPrA the we have the following theorem.

Theorem 1. *Given any SLLD instance, LPrA produces a feasible solution.*

Proof: Assume x_i^l is the output from LPrA. Firstly, it is obvious that x_i^l equals 1 or 0. Next, we will prove x_i^l meet the constraints (2-3).

1. For any $i \in P$, there exist exactly one l^*, that $x_i^{l^*} = 1$. Then $\sum_{l=0}^{l=L} x_i^l = 1$.
2. According to the adjustment process from step 8 to 11 in LPrA, the feasibility of x_i^l will not change before and after this process. Then we only need to prove the solution after the rounding process from step 2 to step 6 is feasible. For any $j \in J$, let

$$P(j) = \{i|a_{ij} = 1, i \in P\}.$$

$$l_i^* = \{l|x_i^l = 1\}$$

Then,

$$\sum_{i\in P(j)} C^{l_i^*} \geq \sum_{i\in P(j)} s_i^* = \sum_{i\in P(j)} \sum_{l=0}^{l=L} C^l x_i^{l*} = \sum_{i\in P} \sum_{l=0}^{l=L} a_{ij} C^l x_i^{l*} \geq d_j. \quad \square$$

Theorem 2. *Given any SLLD instance, LPrA produces an approximation of the optimal solution which satisfies:*

$$z_{LPrA}(SLLD) \leq r * z^*(SLLD) + c_0.$$

Where $z_{LPrA}(SLLD)$ is the value output from LPrA, and r, c_0 is defined by (10) and (11).

Proof: Assume x_i^l is the output solution from LPrA after step 6. Then we have

$$z_{LPrA}(SLLD) \leq \sum_{i\in P} \sum_{l=0}^{l=L} c^l x_i^l.$$

Let's define:

$$P_2 = \{i | s_i^* > C^1, i \in P\} \tag{12}$$

$$P_1 = \{i | 0 < s_i^* \leq C^1, i \in P\} \tag{13}$$

$$P_0 = \{i | s_i^* = 0, i \in P\} \tag{14}$$

Where $s_i^* = \sum_{l=0}^{l=L} C^l x_i^{l*}$. Then,

$$z_{LPrA}(SLLD) \leq \sum_{i \in P} \sum_{l=0}^{l=L} c^l x_i^l = \sum_{i \in P_1} c^1 x_i^1 + \sum_{i \in P_2} \sum_{l=2}^{l=L} c^l x_i^l \leq |P_1| c^1 + \sum_{i \in P_2} c^{l_i^*} x_i^{l_i^*}$$

$$\leq |P| c^1 + \sum_{i \in P_2} \frac{c^{l_i^*}}{c^{l_{(i^*-1)}}} \sum_{l=0}^{l=L} c^l x_i^{l*} \leq c_0 + r * \sum_{i \in P_2} \sum_{l=0}^{l=L} c^l x_i^{l*} \leq r * z^*(SLLD) + c_0. \quad \square$$

4 Conclusion

In this paper, we have studied the storage location and capacity determination problem in emergency resource reserve system, in which a reserve system is set up for holding disaster relief resources, and different level of storage capacity has different construction cost. We first formulate a storage location and capacity determination model to minimize the construction cost. Then, an approximation algorithm is presented by introducing LP-rounding technique. Finally, the proofs of correctness and approximation ratio are given.

For future research, we need to do simulations to demonstrate average performance of the presented algorithm. And it is desirable to develop new techniques to solve the presented model. Finally, uncertainty of the requirement in SLLD also deserve research efforts.

References

[1] G. Barbarosoglu and Y. Arda. A two-stage stochastic programming framework for transportation planning in disaster response. *Journal of the Operational Research Society*, 2004, 55:43-53.

[2] B. M. Beamon and S. A. Kotleba. Inventory modeling for complex emergencies in humanitarian relief operations. *International Journal of Logistics: Research and Applications*,2006, 9(1):1-18.

[3] L. Cooper, Location-allocation problems. *Operational Research*, 1963, 11:331-344.

[4] A. Fiedrich, F. Gehbauer and U. Rickers. Optimized resource allocation for emergency response after earthquake disasters. *Safety Science*, 2000, 35:41-57.

[5] A. Haghani and S. Oh. Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations. *Transportation Research A*, 1996, 30(3):231-250.

[6] A. Kukreja, C. P. Schmidt and D. M. Miller. Stocking decisions for low-usage items in a multilocation inventory system. *Management Science*, 2001, 47:1371-1383.

[7] R. Logendran and M. P. Terrell. Uncapacitated plant location-allocation problems with price sensitive stochastic demands. *Computers and Operations Research*, 1998, 15:189-198.

[8] L. K. Nozick and M. A. Turnquist. Inventory, transportation, service quality and the location of distribution centers. *European Journal of Operational Research*, 2001, 129(2):362-371.

[9] J. Y. Potvin, Y. Xu and I. Benyahia. Vehicle routing and scheduling with dynamic travel times. *Computers and Operations Research*, 2006, 33:1129-1137.

[10] J. B. Sheu. Challenges of Emergency Logistics Management. *Transportation Research Part E*, 2007, 43(6):655-659.

[11] C. Toregas, R. Swain, C. ReVelle and L. Bergman. The location of emergency service facilities. *Operations Research*, 1971, 19(6):1363-1373.

[12] M. L. Wen and K. Iwamura. Facility location-allocation problem in random fuzzy environment: Using (α, β)-cost minimization model under Hurwicz criterion, *Computers and Mathematics with Applications*, 2008, 55(4):704-713.

[13] W. Yi and L. Ozdamar. Fuzzy modeling for coordinating logistics in emergencies. *International Scientific Journal of Methods and Models of Complexity-Special Issue on Societal Problems in Turkey*, 7(1), 2004.

[14] J. Zhou and B. Liu. Modeling capacitated location-allocation problem with fuzzy demands. Technical Report, 2004.