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Optimum Design of Storage Cycle Time Analysis using Queueing Theory and Taguchi Method for a Conveyor and a Rotary Rac

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Abstract This paper analyze the relationship between the cycle time and the configuration factors of a rotary rack system consisting of a rotary rack and a conveyor, by using the cycle time as our criteria for evaluation. Using the queue theory of an M/G/1 model, which has a simpler theoretical formula than that of a GI/G/1 model, to compare the cycle time between the two models. The performance of rotary rack systems are evaluated design of experiment to find the significant factor, by analyzing the inter-relationship between the conveyor speed, the vertical and horizontal speeds of the rotary rack, thresholds, and the arrival rate using an L18 orthogonal array. Next, the factors that affect the cycle time of rotary rack systems are analyzed by using the Taguchi method to find the optimal factor and analyze its sensitivity.

Keywords Materials Handling; Queueing Theory; Taguchi Method; Design of Experiment

1 Introduction

Rotary racks and conveyors are typical material handling equipment in the production and distribution fields. In particular, they are widely used in production factories as a temporary storage system for work in progress. A rotary rack is a multi-layer storage system which rotates horizontally to allow the storage of parts or work in progress and to retrieve them efficiently and speedily. In addition, the rotary rack system is also capable of transferring parts or work in progress, so that there is no need for lots of workers to perform sorting operations, and it may be used to adjust the production speed. Y.J. Hsieh, S.C.Chang and S.E. Chang[1] studied the optimal configuration of rotary racks using the storage and retrieval (S/R) model and the queuing theory of a GI/G/1 model. In their study, they proposed a performance evaluation of the expected waiting time in rotary racks and conveyors. They also analyzed the cycle time of rotary rack systems using a configuration obtained by determining the optimal shape factors of rotary racks in a section and a configuration obtained by an approximate solution. However, the GI/G/1 model formula suggested by them and those cited in the paper are so complicated that it is difficult to use them for actual applications. On the other hand, it is necessary to understand the characteristics of the factors used to optimize the rotary rack system and perform a sensitivity analysis to determine which factors affect the cycle time in a quantitative manner. In this study, we examined the relationship between the cycle time and the shape factors of a rotary rack system consisting of a rotary rack and a conveyor, by using the cycle time as our criteria for evaluation. This study applied the queue theory of an M/G/1 model, which has a simpler theoretical formula than that of a GI/G/1 model, to compare the cycle time between the two models. For this reason, the service time of the rotary rack in terms of horizontal and vertical movements is analyzed by using shape factor b, obtained from the algorithm that obtains a section, proposed by Y.J. Hsieh and S.E. Chang [1]. In addition, an approximate solution for b is also obtained using a heuristic solution. Then, using the approximate solution for b, the GI/G/1 model and the cycle time from the M/G/1 model are compared. Finally, the performance of rotary rack systems is evaluated experimentally to find the significant factor, by analyzing the inter-relationship between the conveyor speed, the vertical and horizontal speeds of the rotary rack, thresholds, and the arrival rate using an L_{18} orthogonal table. Next, the factors that affect the cycle time of rotary rack systems are analyzed by using the Taguchi method to find the optimal factor and analyze its sensitivity.

2 Storage Cycle Time Analysis

2.1 Conveyor and Rotary Rac Model Analysis

The system configuration is shown in Figure 1. Loads transferred from the conveyor through the loading station are stored on the rotary rack. Loads arrive at the rotary rack GI/G/1 model successive arrivals and M/G/1 model arrivals with Poisson process. Each load to be stored is placed at the end of the queue at the loading station before being removed from the conveyor. If loads fall out of the other end of the queue, it will be loaded back onto the conveyor automatically.

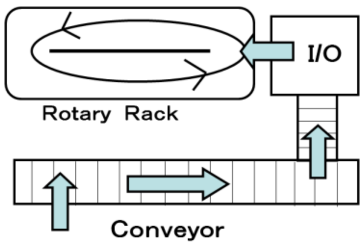

Figure 1.Storage systems of conveyor and Rotary Rack

The conveyor moves in a single direction. It is divided into sections (or spaces), for carrying individual articles. It is assumed that all the sections are of the same size. The conveyor speed is constant and preset by the user in advance. Once loads are loaded onto the conveyor, it moves toward the queue waiting for the rotary rack to pick it up. The storage position of each load will be decided randomly when it joins the queue waiting for the rotary rack (a random storage system is used).

The rotary rack is an independent multi-level storage device which has trays rotating in the horizontal direction. It also has a storage/retrieval (S/R) system which is intended to store and retrieve articles between the rotary rack and conveyor. The I/O points of the S/R device are located at the bottom right side in the figure 1. Every storage and retrieval operation starts and ends at the I/O point. Storage and retrieval are performed on a first-in-first-served (FIFS) basis. The S/R device only handles a single article at a time. The S/R device moves vertically while the rotary rack moves horizontally, synchronized with each other. The length (l) and height (h) of the system, S/R device speed (Vv: vertical motion) and rotary rack speed (Vh: horizontal motion) are preset by the user in advance.

2.2 Calculation of S/R and GI/G/1 Model

The waiting time is modeled using a queue. Here, the cycle time of the whole system is represented by the formula.

$$E[ST] = E[W_C] + E[S_C] + E[W_r] + E[S_r] \tag{1}$$

Formula (1) consists of the following four elements. The duration of each element is as follows:

$E[W_C]$:Waiting time on the conveyor ; $E[S_C]$: Conveyor service time

$E[W_r]$:Waiting time on the rotary rack ; $E[S_r]$: Rotary rack service time

The method for calculating the duration of each element is as follows:

2.2.1 Waiting time on the conveyor

The waiting time on the conveyor is given by equation 2, from queuing theory.

$$E[W_C] = \rho_C D + (1-\rho_C)\bar{D} \tag{2}$$

where D is the waiting time of a queue under a certain condition, $\bar{D}$ is the waiting time without a queue, and ρ_C is the utilization factor of the conveyor. Then, D can be derived as follows:Let R be the waiting time of an article at the beginning of a queue, τ is the conveyor cycle time, and L is the number of articles present before the article that has just arrived. D is provided by the following equation.

$$D = R + \tau L \tag{3}$$

$\bar{D}$ and R are constant when the conveyor cycle time is (0,1/V), where V is the speed of conveyor. The waiting time can be given as follows:

$$\bar{D} = R = 1/2V \tag{4}$$

The conveyor waiting time E[W_C] is given as next equation.

$$E[W_C] = 1/(2(V-\lambda)) \tag{5}$$

2.2.2 Expected travel time of the conveyor

The expected travel time of the conveyor can be obtained using this equation .

$$E[S_C] = N / V \tag{6}$$

where, N: the number of conveyor windows between the loading station of the conveyor and the rotary rack.; V: is the speed of conveyor and the cycle time is 1/V.

2.2.3Service time of the rotary rack

In calculating the rotary rack service time, s is vertical lifting time of the S/R device from the I/O point to the top level and t is horizontal rotating time of the rotary racfk for half of the rack;X is rack rotation time from a random point on the rack to fixed position; and Y=S/R device travel time from the I/O point to random point.t Here, (X,Y) is the storage position of an article requested by the user. The storage positions (X,Y) of articles are distributed evenly between 0≦X≦t and 0≦Y≦s. Generally speaking, it is said that s must be less than or the same as t, as far as the rotary rack is concerned .If X～uniform(0,t), Y～uniform(0,t) and Sr=max[X+Y]+Y. The expected rotary service time,E[Sr] ,is given by Ross[2] and (Y.J.Hsieh.,etal)

$$E[S_r] = \frac{t+s}{2} + \frac{s^2}{6t} \tag{7}$$

2.2.4 Waiting time on the rotary rack

The waiting time of rotary rack E[W $_r$] is given by Gross and Harris .The waiting time on the rotary rack can be expressed by the following equation, based on the GI/G/1 model queuing theory:

$$E[W_r] \cong \left(\frac{C_a^2 + C_e^2}{2}\right)\left(\frac{\rho_r}{1-\rho_r}\right) E[S_r], \tag{8}$$

where, S $_r$ is Rotary rack utilization rate ,and C $_a^2$ 〔C_e^2〕 :is the squared coefficient of variation ,of the inter-arrival(service cycle)time at the rotary rack; $\rho_r = \lambda$ E[S $_r$] is the utilization of the rotary rack. Coefficient of variation of the device handling time.The derivation of C_a^2 is given by (Bozer and Hsieh[3]).

$$C_a^2 = \frac{(V-\lambda)[2\lambda - VF(1/V)]}{V^2 F(1/V)} \tag{9}$$

$$C_e = \frac{3t^4 - 3t^2s^2 + 9ts^3 - s^4}{9t^4 + 18t^3s + 15t^2s^2 + 6ts^3 + s^4} \tag{10}$$

2.3 Calculation of M/G/1 Model

The calculation of GI/G/1 model becomes complicated. Hsieh etal[1] propose the three approximation equations for GI/G/1 model. However, the equation is complicated. Therefore, GI/G/1 model is calculated approximately in the M/G/1 model. By the numerical calculation, the comparison of error between the M/G/1 model and M/G/1 model are computed.

In this study, the waiting time on the rotary rack is also verified by using the M/G/1 model queuing theory. The formula for calculating the waiting time on the rotary rack using an M/G/1 model is as follows:

$$E[W_r]=\left(\frac{1+C_e^2}{2}\right)\left(\frac{\rho_r}{1-\rho_r}\right)E[S_r] \tag{11}$$

2.4. Rotary Rack Design

The optimum shape of the rotary rack is the expected storage cycle time becomes minimize. The shape factor b is standardized as b=s/t, where $0 \leqq b \leqq 1$, in order to facilitate the analysis of the optimal rotary rack design(Bozer and White [4],Han et al.[5]). By using b, the formula for the rotary rack service time $E[S_r]$ can be converted as follows:

$$E[S_r]=\left(\frac{1+b}{2}+\frac{b^2}{6}\right)\sqrt{\frac{A}{bV_vV_h}} \tag{12}$$

where,V_v: Vertical speed of the S/R device.V_h: Horizontal speed of the rotary rack:A=lh: The volume of the rotary rack (rack length: l, rack height: h.From equation (12),the shape factor is computed as $b=(\sqrt{5}-1)/2$ give as Y.J.Hsieh et al.[1]. Stability and feasible range of shape b must be checked in order to obtain the near optimum shape. After checking the steady state of the system and the range in which b can be used, the actual design of the rotary rack is derived heuristically. The algorithm for deriving an approximate solution of the configuration factor is shown below[1].

Step 1: Setting the initial values of λ ,V,N, ε 、 A,V_v, and V_h and $b^{(i)}=\mathrm{Max}(b_1,0)$

First, set the initial values including the conveyor speed.

Step 2: Use b(i) to compute E[ST](i) from equation (1).

Step 3: Set $b^{(i+1)}=b^{(i)}+\delta$,update $E[ST]^{(i+1)}$from equation(1)

If $|E[ST]^{(i+1)}-E[ST]^{(i)}| \leqq \varepsilon$,then set $b^*=b^{(i+1)}$ and b* is near optimum shape factor, stop;

else

Set i=i+1,and repeat Step3.

4. Taguchi method

4.1 Inner Noise and Outer noise

The design of rotary rack must be decide the parameter of speed for the conveyor, horizontal and vertical velocity of rotary rack and shape factors. Recently, the parameter methods are used for the design of machine using the no trial of machine design. The prototype becomes useless. Taguchi method is a method for designing the parameter in the theoretical formula and simulation regardless the experiment[5]. The parameter of rotary rack is designed using the theoretical equation (1).The control factors are the conveyor speed, vertical velocity, horizontal velocity, threshold and arrival rates.Table 1 shows 3 levels of control factors. The level value of the Table 1 set 20% of the initial value of control factors with the second level.

Level value of the control factors are allocated in L18 orthogonal table. This is the allocation of the inside. The slippage of speed and threshold and arrival rate are made to be 2%of the central value.

Table 1. 3 levels of control factors

Factors	Level 1	Factor 2	Factor3
Conveyor speed	Level2 × 0.8 m/s	1m/s	Level2 × 1.2
RR vertical speed	Level2 × 0.8 m/s	1m/s	Level2 × 1.2
RR horizontal speed	Level2 × 0.8m/s	1m/s	Level2 × 1.2
Threshold	Level2 × 0.8 m/s	0.001	Level2 × 1.2
Arrival rates	4	6	8

This value also allocated in L_{18} orthogonal table. This is the outside allocation. The L_{18} orthogonal table is computed from signal noise method.

Table 2. 3 levels of the error factors

Factors			
Conveyor speed	0.98	1	1.02
RR vertical speed	0.98	1	1.02
RR horizontal speed	0.98	1	1.02
Threshold	0.98	1	1.02
Arrival rates	0.98	1	1.02

5. Numerical Results

5.1 Comparison of the cycle time

Table 3 shows the comparison of GI/G /1 model and M/G/1 model. The cycle time using the GI/G/1 model and the cycle time using a M/G/1 model are very close to each other. It can be found that the cycle time of the whole system rapidly increases as the arrival rate λ increases.

Table 3 Comparison of cycle time between M/G/1 and GI/G/1 model

λ	b *(M/G/1)	E[ST](M/G/1)	E[ST](GI/G/1)
3	0.582	19.722	19.718
4	0.593	21.085	21.183
5	0.603	23.212	23.183
6	0.611	27.007	26.937
7	0.617	35.701	35.516
8	0.619	76.199	75.417

The analysis of variance of the orthogonal array is computed to analyze the characteristics of this system. The results are shown in Table 4 .The analysis of variance of the orthogonal array revealed that the conveyor speed , the vertical and horizontal speeds of the rotary rack have a significance level of 1 %. The relationship between each factors and its sensitivity are shown in Figure 1. The sensitivity of the conveyor speed and the vertical and horizontal speeds of the rotary rack all increased.

Table 4 Analysis of variance

Source	SS	df	V	F
Conveyor speed	400.5	2	200.25	24.33**
RRvertical speed	184.9	2	92.46	11.24**
RRhorizontal speed	186.0	2	92.99	11.30**
threshold	14.4	2	7.16	0.87
Arrival rate	19.9	2	9.93	1.21
A×B	30.4	4	7.60	0.92
A×C	30.0	4	7.51	0.91
B×C	146.2	4	36.55	4.44
e	32.9	4	8.23	
Total	1045.21	26		

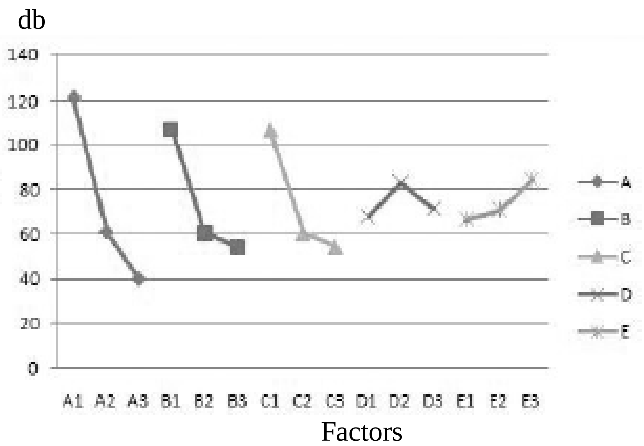

Figure 2 . Sensitivity analysis between factors

5.2 Shape factor and queueing model

Assuming a configuration factor of b=0.618, the cycle time at b* and the cycle time at b were compared using an arrival rate of λ. The configuration factor b was very close to b* at every arrival rate λ, with an error of 1.5s. The value b* was not affected by λ. It is found from this fact that it is not necessary to change the rack design even if the request for the throughput from the user changes.In reality, it is difficult to change the rotary rack design if a user requests more throughput. For this reason, the configuration factor b suggested here seems to meet such a user request quite well, although it does not provide a completely optimal solution.

The cycle time of the M/G/1 model is very close to that of the GI/G/1 model, so it seems possible to adopt the M/G/1 model, which is simpler than the GI/G/1 model.

The analysis of variance of the orthogonal array using Taguchi's method showed that the conveyor speed and the vertical and horizontal speeds of the rotary rack have a significance level of 1%. From this result, it can be understood that these three factors have a large effect on the rotary rack system. In addition, it was found that the vertical speed and horizontal speed of the rotary rack have the same degree of influence. The relationship between the factors shown in Figure 5 and their sensitivity also demonstrated that these three factors have a large effect on the rotary rack system.

6. Conclusions

This paper summarized the results of recent research[1] about the rotary rack system and clarified the present level of research. It also optimized the cycle time of the rotary rack system consisting of a rotary rack and a conveyor, by making calculations using GI/G/1 and M/G/1 queuing theory models. A rotary rack design that is both flexible and efficient was proposed using a heuristic method. Shape factor b is analyzed, and then a specific shape factor b is proposed which is almost unaffected by the arrival rate λ and does not require a change in the rack design. In addition, the sensitivity of the factors and elements is analyzed using Taguchi's parameter design method. The analysis of variance of the orthogonal array is computed to analyze the characteristics of this system.

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